A Determination of Earth Equatorial Ellipticity from Seven

Months of Syncom II Longitude Drift

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The 24-hour Syncom II satellite has been under periodic observation

by range and range rate radar and Minitrack Radio Interferometer stations

since mid-1963 [Wagner, 1964b]. Seven months of longitude drift in the

vicinity of two momentarily stationary configurations were analyzed for

sensitivity to hypothetical longitude components of the earth's gravity

field which would be in "resonance" on such a satellite [Blitzer et al.,

1962; Wagner, 1964a]. This drift, in the region 54 to 64 degrees west

longitude, was derived from orbits calculated at the Goddard Space

Flight Center.

From mid-August 1963 to late November 1963, the Figure 8 ground

track of Syncom II drifted from 55° west to 59° west, with a mean

acceleration of

$$-(1.27 \pm .02) \times 10^{-3} \text{ degrees/day}^2. \tag{1}$$

The average growth of the semi-major axis for this period was estimated as

$$(.0993 \pm .0042) \text{ km/day}.$$

The Figure 8 configuration was momentarily stationary at about 54.76° on

6 September 1963, at which time the semi-major axis was

$$(42166.0 \pm .2) \text{ km}.$$

On 28 November 1963, the westward drift of Syncom II was stopped by

ground command firing of tangentially oriented cold gas jets on-board

the satellite.

From early December 1963 to mid-February 1964, the Figure 8 ground

track of Syncom II drifted from a momentarily stationary position at

about 59.15° west, (semi-major axis: $42165.9 \pm .4$ km) to 63.5° west.

The estimated mean geographic longitude acceleration of the ground track

for this period was

$$-(1.32 \pm .02) \times 10^{-3} \text{ degrees/day}^2. \tag{2}$$

The average growth of the semi-major axis for this period was

$$(.0994 \pm .0080) \text{ km/day.}$$

Simulated Syncom II trajectories for these drift periods, starting with

the initial orbital elements, show good agreement with the observed

trajectories if an earth gravity field with longitude dependence is

used in the particle program of the simulation. Sun and moon influence, as well

as earth zonal gravity influence, on drift acceleration (as assessed by

this simulation) for the full seven months of Syncom II data analyzed,

appears to be negligible compared to hypothetical earth gravity with

longitude dependence. Other possible causes of this observed long term

accelerated longitude drift, such as (1) selective outgassing or leaking

of on-board gas jets, (2) micrometeorite collisions, (3) solar wind or

radiation interactions, or (4) geomagnetic field interactions in the

environment of Syncom II all appear to be extremely unlikely.

The theory in Wagner [1964b] showed that any ellipticity of the

earth's equator will cause an otherwise stationary Figure 8 ground

track of a 24-hour inclined near circular orbit satellite, to drift in

longitude with an acceleration given by

$$\ddot{\lambda} = -A_{22} \sin 2\gamma, \tag{3}$$

where

$$\Lambda_{22} = -72\pi^2 J_{22}(R_o/a)^2 \frac{(\cos^2 i + 1)}{2}. \qquad \text{rad/sidereal-day}^2 \qquad (4)$$

[Relations (3) and (4), without the inclination factor, are also to be found in Allan [1963] and Wagner [1964a].]

γ is the nodal longitude of the 24-hour configuration east of the minor axis of the elliptical equator. J_{22} is the amplitude of the first significant longitude dependent term in the spherical harmonic expansion of the earth's gravity potential. It is related to the difference between major and minor earth equatorial radii by: [Izsak, 1961; Wagner, 1962; Kaula, 1965]

$$a_o - b_o = -6R_o J_{22}.$$

R_o is the mean equatorial radius of the earth. a is the semi-major axis of the 24-hour satellite. "i" is its inclination. For Syncom II, the inclination during both drift periods analyzed was close to 33.0^o.

Under the assumption that the above observed drift acceleration is sensing only the lowest order of longitude dependent earth gravity (that associated with the ellipticity of the earth's equator), the two drift accelerations of Syncom II, Equations (1) and (2), satisfy relations (3) and (4) uniquely with the following values of equatorial ellipticity:

$$J_{22} \text{ (unadjusted for higher order} = -(1.70 \pm .05) \times 10^{-6} \qquad (5)$$
$$\text{earth gravity effects)}$$

corresponding to a difference in major and minor equatorial radii of:

$$a_o - b_o = 65 \pm 2 \text{ meters}$$

and,

$$\lambda_{22} = (19 \pm 6) \text{ degrees west of Greenwich,} \qquad (6)$$

locating the major axis of the earth's elliptical equator.

From Table 1, it is seen that these values for the second order tesseral harmonic of earth gravity are in reasonable agreement with recent determinations of longitude gravity from lower altitude satellite observations and surface gravimeter data. From a consensus of recent and older geoids which give tesseral field coefficients to higher order than the second, it would appear that at the high altitude of the 24-hour satellite, the second order longitude effect accounts for about 85% of the full field effect at $54°$ to $64°$ west longitude. The full longitude field is consistently depressed below the J_{22} field at these longitudes for all the recent geoids of Table 1. On the basis of the recent measures of higher order tesseral gravity, calculations show the Syncom II estimates (5) and (6) should be:

$$J_{22} \text{ (adjusted for probable higher order earth gravity effects)} = -(1.9 \pm .2) \times 10^{-6} \qquad (7)$$

or,

$$a_o - b_o = 73 \pm 8 \text{ meters}$$

and, $\qquad \lambda_{22} = (21 \pm 7)°$ west of Greenwich. $\qquad (8)$

That longitude dependent earth gravity exists seems well established from a large number of recent gravity reductions on different bases (Table 1). This 24-hour satellite reduction (because the altitude is so high) appears to separate out the second order effect almost entirely from the sum of all higher order earth gravity effects. Unless the earth is far more inhomogeneous in longitude than is thought to date, (7) and (8) are to be considered absolute estimates of the bounds on the ellipticity of the equator.

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